ONSET OF CAVITATION DURING FLOW AROUND A PROTUBERANCE ON A WALL IN A PLANAR CHANNEL

L. A. Galin, A. P. Frolov,
and K. K. Shal'nev

Prediction of the conditions for cavitation onset and thus cavitative erosion is quite important for designing and using hydrotechnical equipment and various types of hydraulic machines. The problem of determining the cavitation sensitivity of an airfoil was reduced to an integral Fredholm equation of the second kind in [1]. It was suggested there that cavitation arises at that part of the profile where the fluid velocity reaches its maximum value. It is also known [5] that conditions for the onset of cavitation are governed by both the geometric characteristics of the object and by the physical properties of the liquid (viscosity, surface tension, amount of dissolved gas, etc.). Below we take up the problem of the flow around a protuberance on the wall of a planar channel, determining the dependence of the critical number for cavitation onset on the protuberance height, the channel height, and the slope of the leading face of the protuberance. All the physical properties of the liquid are assumed constant, so only the geometric characteristics of the protuberance change.

We assume that the flow around a corner is of a local nature and that flow does not occur around an infinitely sharp protuberance. Actually, a constant-pressure separation line forms immediately beyond a corner in a liquid flow, and the radius of curvature of this line does not depend or does not strongly depend on the protuberance geometry. We thus assume that the radius of curvature of a streamline near a corner, unknown beforehand, remains constant while the other flow characteristics change.

This assumption is justified below by a comparison of the calculated and experimental results.

1. Using the model of [2] for the flow of a cavitating liquid at the initial stage of the development of cavitation, we can determine the size of the seat of cavitative erosion for flow of a liquid around a polygonal object. The size of the seats of erosion predicted by this theory turns out to be in good agreement with experiment. However, since the contour of the object is constructed from straight line segments, the cavitation which arises near corners protruding into the flow should theoretically disappear only at an infinite cavitation number

$$\varkappa = \frac{2(p - p_v)}{\rho v^2} \tag{1.1}$$

where p, ρ, v, and p_v are the pressure, density, velocity, and vapor pressure of the liquid, respectively. This theoretical prediction is contradicted by experimental data showing finite critical numbers for the onset and disappearance of cavitation during flow of a liquid around a polygonal object. This discrepancy is attributed primarily to the difference between the geometry of a real flow and the model of separation-free flow around a corner. Actually, the liquid immediately beyond the corner is separated from the surface, and a region filled with a relatively slowly moving liquid in eddying motion is formed.

To determine the flow structure near a protuberance, we carried out experiments in a hydraulic pan with a rectangular protuberance 40 mm high; the channel width was 100 mm. At a flow velocity of V = 0.12 m/sec, the Reynolds number N_{Re} corresponding to the protuberance height was ~5000. The water in the pan was dyed with black ink, and the flow was emphasized by a fine aluminum powder floating on the water surface.

Moscow. Translated from Zhurnal Prikladnoi Mechaniki i Tekhnicheskoi Fiziki, No. 4, pp. 172-175, July-August, 1970. Original article submitted December 15, 1969.

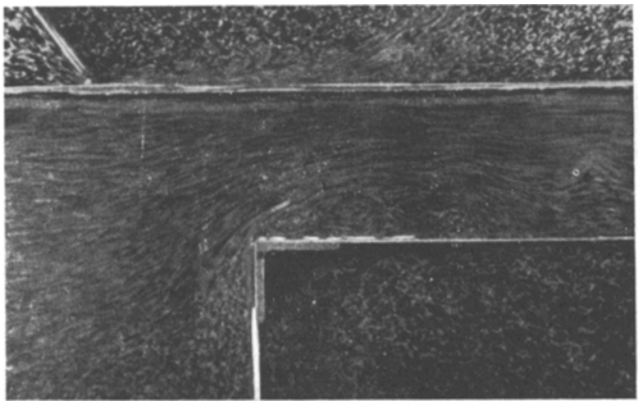

Fig. 1

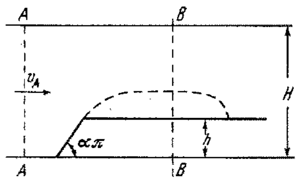

Fig. 2

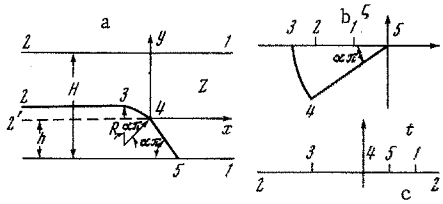

Fig. 3

Visual observation and photographs of the flow show that the flow around the protuberance involves the formation of free eddies filling the separated-flow region. The boundary of the main flow in the eddy region is essentially steady and is clearly observable (Fig. 1). In the experiments whose results are described below, carried out in a study of the onset of cavitation at a protuberance in the working chamber of a hydrodynamic tube, the N_{Re} values were approximately two orders of magnitude greater than in the hydraulic pan. However, as the dependence of the Strouhal number on the Reynolds number shows, the eddy structure of the separation zone for a bluff object [3, 4] is retained during flow around the model in the tube. As N_{Re} increases, the eddies decrease in size, and the boundary separating the two flow regions becomes smoother.

In discussing flow around a protuberance at the bottom of a planar channel, we neglect diffusion of the eddy motion from the separation zone into the main potential flow, and we assume the boundary between these regions to be a rigid, impenetrable wall. For a comparatively short distance along the channel, between cross sections A-A and B-B (Fig. 2), this assumption holds quite well. Then, using conformal mapping, we consider an infinite channel at whose ends, 1-1 and 2-2 (Fig. 3), the same hydrodynamic conditions hold as in cross sections A-A and B-B (Fig. 2), respectively. We assume that the flow between the separation region and the potential flow consists of a semiinfinite rectilinear segment 2-3 and a curvilinear region 3-4, whose shape is to be determined. The other flow boundaries are formed by the rigid, impenetrable walls of the channel.

2. On the basis of the above discussion, we determine the conditions for the onset of cavitation during flow of an ideal incompressible liquid around a protruding step having a rounded edge. Figure 3 shows the potential flow on the plane of the complex variable $z = x + iy$. At the boundaries of the flow region we have the following conditions.

Cavitation bubbles form and grow at edge 3-4 earlier than anywhere else in the flow. As in [2], we assume that at that part of the flow boundary at which cavitation bubbles arise, the pressure is constant and equal to the vapor pressure.

We take as the critical condition for the onset or disappearance of cavitation the situation in which the cavitation zone covers entire edge 3-4, i.e.,

$$p = p_v \text{ at } 3\text{-}4. \tag{2.1}$$

For a relatively small radius of curvature of edge 3-4, this definition of the critical cavitation number is approximately equal to the cavitation number determined experimentally from the acoustic emission

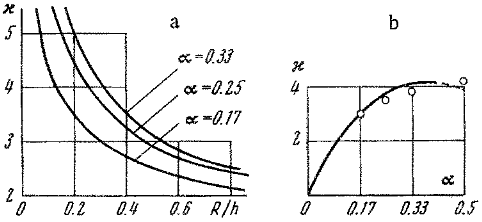

Fig. 4

of the liquid and from the corresponding appearance of individual cavitation bubbles at some point of region 3-4.

In the plane of the hodograph $\zeta = dw/dz$, where w (z) is the complex flow potential, the real-flow region 1-2-3-4-5-1 corresponds to the sector of a circle of radius v_3 subtended by an angle $\alpha\pi$ (Fig. 3b). Here v_3 is the liquid velocity at point 3. To solve the problem we conformally map the real-flow region on the z plane into this sector on the ζ plane. We first conformally map sector 1-2-3-4-5-1 on the ζ plane into the upper half-plane of the auxiliary complex variable t (Fig. 3c) by means of the function

$$t = \frac{u-1}{(u_2-u)(u_2-1)}, \qquad u = \frac{1}{2}\left[\left(\frac{\zeta}{v_3}\right)^{1/\alpha} + \left(\frac{v_3}{\zeta}\right)^{1/\alpha}\right]. \tag{2.2}$$

Here we have

$$t_2 = \infty, \qquad t_3 = \frac{2}{1-u_2^2}, \qquad t_4 = 0, \qquad t_5 = \frac{1}{1-u_2},$$

$$u_1 = -\frac{1+(\varkappa+1)^{1/\alpha}}{2(\varkappa+1)^{1/2\alpha}}, \qquad v_3^2 = v_1^2(\varkappa+1).$$

The constant u_2 is determined in the course of the solution.

The t upper half-plane is mapped conformally onto the polygon 1-2-2'-4-5-1 of the z plane by means of the Christoffel-Schwartz integral

$$z = -c\int_0^t \frac{t^\alpha dt}{(t-t_5)^\alpha (t-t_1)} \tag{2.3}$$

Using an approximate conformal mapping, we convert the t upper half-plane into the curvilinear polygon 1-2-3-4-5-1 in the z plane, using the function

$$z = -c\int_0^t \frac{t^\alpha + \gamma(t-t_3)^\alpha}{(t-t_5)^\alpha (t-t_1)}\, dt, \tag{2.4}$$

where the constants c and γ are governed by the geometric dimensions of the curvilinear polygon. Integral (2.4) determines the shape of edge 3-4 for $t_3 \le t \le 0$.

We circumvent point 1 on the t plane along an infinitesimally small semicircle, and we circumvent point 2 along an infinitely large semicircle. From Eq. (2.4) we find two equations for the unknown constants:

$$H = c\pi\frac{t_1^\alpha + \gamma(t_1-t_3)^\alpha}{(t_1-t_5)^\alpha}, \qquad H - h - R(1-\cos\alpha\pi) = c\pi(1+\gamma), \tag{2.5}$$

where R is the characteristic radius of curvature of edge 3-4. The other equations necessary can be found from the correspondence of points 3 on the t and z planes; from Fig. 3 we see that

$$z_3 = -R\sin\alpha\pi - iR(1-\cos\alpha\pi) \tag{2.6}$$

Substituting Eqs. (2.2) and (2.6) into Eq. (2.4), we find two other real equations for the unknowns c, γ, u_2, and R as functions of the critical cavitation number $\varkappa$:

$$R\sin\alpha\pi = cf + c\gamma g\cos\alpha\pi, \quad R(1-\cos\alpha\pi) = c\gamma g\sin\alpha\pi$$

$$f = \int_0^\beta \frac{x^\alpha dx}{(x+t_5)^\alpha (x+t_1)}, \qquad g = \int_0^\beta \frac{(\beta - x)^\alpha dx}{(x+t_5)^\alpha (x+t_1)}, \qquad \beta = -t_3. \tag{2.7}$$

System (2.5), (2.7) can be reduced to a single equation for u_2:

$$\frac{\pi(a_1-a_2)R(1-\cos\alpha\pi)}{hf\sin\alpha\pi} - \frac{H}{h} + \left[\frac{H}{h} - 1 - \frac{R}{h}(1-\cos\alpha\pi)\right]a_2 = 0,$$

$$\frac{R}{h} = \frac{(H/h-1)(a_1+a_2 f/g) - (H/h)(1+f/g)}{(1-\cos\alpha\pi)(a_1+a_2 f/g)}, \tag{2.8}$$

$$a_1 = \left(\frac{t_1}{t_1-t_5}\right)^\alpha, \qquad a_2 = \left(\frac{t_1-t_3}{t_1-t_5}\right)^\alpha.$$

The integrals f and g in Eq. (2.8) can be expressed in terms of elementary functions for rational α values. For practical calculations, however, it is simpler to calculate them directly numerically.

Solving Eq. (2.8) on a computer, we find the dependence of the critical cavitation number on the characteristic radius of curvature R of the edge. The results calculated for several α values and for $h/H = 0.2$ are shown in Fig. 4a. The curves here have the same feature; i.e., as $R \to 0$, $\varkappa \to \infty$. However, this feature does not contradict the experimental data showing finite critical cavitation numbers during flow around corners, since separation-free flow around a sharp edge ($R \sim 0$) can occur only at very small N_{Re} ("creeping flow"), quite uncharacteristic of cavitating flow. Assuming R = const over some α range and using the data of Fig. 4a, we can construct the $\varkappa = \varkappa(\alpha)$ dependence (Fig. 4b).

3. An experimental study was made of the $\varkappa = \varkappa(\alpha)$ dependence for flow around a protuberance in the working chamber of a hydrodynamic tube. The working chamber has a cross section of 20×100 mm, and the protuberance height is 20 mm. Four models having angles $\alpha = 0.50$, 0.33, 0.25, and 0.17 were tested. The liquid velocity before reaching the model was $v \approx 14.2$ m/sec, the same in all the experiments.

The onset of cavitation was determined by an acoustic method, from the appearance of the characteristic clicking noise. To eliminate the error in the critical cavitation number $\varkappa$ arising because of "hysteresis," we carried out "forward" and "reverse" experiments, by reducing the pressure in the equalizing reservoir of the tube and by increasing it until a certain cavitation state, marked by the appearance of the characteristic noise, was reached. The cavitation number was determined from (Fig. 2)

$$\varkappa = \frac{p_A - p_T - p_v}{q}, \qquad q = 0.5\rho v_A^2, \tag{3.1}$$

where p_T is the atmospheric pressure and p_v is the vapor pressure.

The points in Fig. 4b show the experimental critical cavitation numbers $\varkappa = \varkappa(\alpha)$ determined. A comparison of the experimental and calculated $\varkappa$ values shows good agreement for $\alpha \leq 0.4$. For $0.4 < \alpha \leq 0.5$, the theoretical curve deviates from the experimental points, apparently because of a violation of the condition R = const at large α (constancy of R was assumed for the construction of the curve in Fig. 4b).

LITERATURE CITED

1. I. Anton and O. Popa, "Determination of the sensitivity to cavitation of a cascade of hydrofoils of arbitrary shape," Rev. Roum. Sci. Techn. Mec. Appl., 12, No. 2 (1967).
2. L. A. Galin, V. G. Markov, and A. P. Frolov, "A model for cavitation flow during the initial stage of cavitation," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 4 (1969).
3. V. I. Il'ichev, "Frequency of cavitation-bubble formation in turbulent boundary layers and in wakes," Dokl. Akad. Nauk SSSR, 136, No. 5 (1956).
4. S. G. Popov, "Relation between the Strouhal and Reynolds numbers during planar flow around a circular cylinder," Izv. Akad. Nauk SSSR, Mekh. Zhidkosti i Gaza, No. 2 (1966).
5. A. D. Pernik, Problems of Cavitation [in Russian], Sudpromgiz, Leningrad (1963).